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Daniele Pucci[1] Tarek Hamel[2] Pascal Morin[3] Claude Samson[1]

[1]INRIA Sophia-Antipolis Méditerranée
2004 Route des Lucioles BP 93
06902 Sophia-Antipolis, France
name.surname@inria.fr

[2]I3S Université Nice Sophia-Antipolis
2000 Route des Lucioles BP 121
06902 Sophia-Antipolis, France
thamel@i3s.unice.fr

[3]ISIR
Université Pierre et Marie Curie (UPMC)
75005 Paris, France
morin@isir.upmc.fr

***Abstract*— The modeling and control of a class of thrust-propelled aerial vehicles subjected to lift and drag aerodynamic forces is addressed. Assuming a rotational symmetry of the vehicle's envelope about an axis and the alignment of the thrust force with this axis, one shows that the resultant of aerodynamic forces can be decomposed as the sum of a term in the direction of the air velocity and a term in the direction of the thrust force. Conditions allowing for the derivation of a family of models of aerodynamic forces for which the first term does not depend on the vehicle's orientation are pointed out. When such a model applies, pre-compensation of the latter term with the thrust input allows one to recast the control problem into the simpler case of a spherical vehicle subjected to drag only for which nonlinear feedback controllers endowed with strong stability and convergence properties have been reported in prior studies. Beside the adaptation of these control results, the paper extends a previous work by the authors in two directions. First, the 3D case is addressed whereas only motions in a single vertical plane was considered. Secondly, the family of models of aerodynamic forces for which the aforementioned transformation holds is enlarged.**

I. Introduction

Feedback control of aerial vehicles in order to achieve some degree of autonomy remains an active research domain after decades of studies on the subject. The complexity of aerodynamic effects and the diversity of flying vehicles partly account for this continued interest. Lately, the emergence of small vehicles for robotic applications (helicopters, quad-rotors, etc) has also renewed the interest of the control community for these systems. Most aerial vehicles belong either to the class of fixed-wing vehicles, or to that of rotary-wing vehicles. The first class is mainly composed of airplanes. In this case, weight is compensated for by lift forces acting essentially on the wings, and propulsion is used to counteract drag forces associated with large air velocities. The second class contains several types of systems, like helicopters, ducted fans, quad-rotors, etc. In this case, lift forces are usually not preponderant and the *thrust force*, produced by one or several propellers, has also to compensate for the vehicle's weight. These vehicles are usually referred to as Vertical Take-Off and Landing vehicles (VTOLs) because they can perform stationary flight (hovering). On the other hand, energy consumption is high due to small lift-to-drag ratios. By contrast, airplanes cannot (usually) perform stationary flight, but they are much more efficient energetically than VTOLs in cruising mode.

Control design techniques for airplanes and VTOLs have developed along different directions and suffer from specific limitations. Feedback control of airplanes explicitly takes into account lift forces via linearized models at low angles of attack. Based on these models, stabilization is usually achieved through linear control techniques [1]. As a consequence, the obtained stability is local and difficult to quantify. Linear techniques are used for hovering VTOLs too, but several nonlinear feedback methods have also been proposed in the last decade to enlarge the provable domain of stability [2] [3] [4] [5]. These methods, however, are based on simplified dynamic models that neglect aerodynamic forces. For this reason, they are not best suited to the control of aerial vehicles moving fast or subjected to strong wind variations. Another drawback of the independent development of control methods for airplanes and VTOLs is the lack of tools for flying vehicles that belong to both classes. These are usually referred to as *convertible* because they can perform stationary flight and also benefit from lift properties at high airspeed via optimized aerodynamic profiles. The renewed interest in such vehicles and their control reflects in the growing number of studies devoted to them in recent years [6] [7] [8] [9], even though the literature in this domain is not much developed yet. One of the motivations for elaborating more versatile control solutions is that the automatic monitoring of the delicate transitions between stationary flight and cruising modes, in relation to the strong variations of drag and lift forces during these transitions, remains a challenge to these days. A first step in this direction consists in taking into account drag forces that do not depend on the vehicle's orientation [10], as in the case of spherical bodies.

The present paper essentially aims at extending [10] by taking lift forces into account and extending to the 3D case a previous contribution [11] concerning vehicles moving in the vertical plane (2D case) which shows how, for a particular class of models of lift and drag aerodynamic forces acting on a wing, it is possible to bring the control problem back to the simpler one of controlling a spherical body subjected to a drag component solely. One can then apply the nonlinear control schemes proposed in [10] for which quasi global stability and convergence results are established. The results here reported thus constitute a contribution to setting the principles of a general nonlinear control framework that applies to many aerial vehicles evolving in a large range of operational and environmental conditions.

The paper is organized as follows. Assumptions about the shape of the flying body and the means of actuation that are used to control its motion, complemented with notation and recalls of classical dynamics equations, are presented in Section II. The core of the paper's original technical results concerns the modeling of aerodynamic forces acting on bisymmetric bodies and the characterization of a subset of models which simplify the control design. These results are reported in Section III. In order to illustrate the usefulness of these results at the control design level with an example, Section IV gives the adapted version of a velocity control scheme proposed in [10]. The concluding Section V offers complementary remarks and points out research perspectives.

II. Background

A. Body's shape and symmetry assumptions

Shape symmetries of aerial vehicles –as well as of marine and ground vehicles– are not coincidental. Simplification and cost reduction of the manufacturing process, despite their importance, are clearly not the main incentives accounting for the ubiquitous use of symmetric shapes. In this respect, Nature was first to give the example with most of the animals populating the Earth. On the basis of this observation, scientific minds could figure out numerous practical advantages resulting from symmetry properties. However, for flying purposes, not all symmetries are equally interesting. For instance, the sphere which represents the simplest most perfect symmetric 3D-shape is not best suited for energy-efficient long-distance flights because it does not allow for the creation of "magical" lift forces which counteract the effects of gravity, in the same way –and almost as well– as wheel-ground contact reaction forces for terrestrial vehicles, and buoyancy for marine and underwater vehicles. We here consider the next simplest kind of symmetries, associated with ovoid and annular shapes, in order to figure out aerodynamic properties induced by them and their practical interest. The present study thus focuses on vehicles that can be modeled in the first approximation by a single, symmetric body immersed in a fluid which exerts motion reaction forces on it, and whose body surface $\mathcal{S}$ is characterized by the existence of an orthonormal body frame $\mathcal{B} = \{G; \vec{\imath}, \vec{\jmath}, \vec{k}\}$ such that

Assumption 1 *Any point $P \in \mathcal{S}$ transformed by the composition of two rotations of angles θ and π about the axes $G\vec{k}$ and $G\vec{\jmath}$, i.e. by the operator defined by*

$$g_\theta(\cdot) = (rot_{G\vec{k}}(\theta) \circ rot_{G\vec{\jmath}}(\pi))(\cdot),$$

also belongs to $\mathcal{S}$, i.e.

$$g_\theta(P) \in \mathcal{S},$$

where $rot_{O\vec{v}}(\xi)(P)$ stands for the rotation about the axis $O\vec{v}$, by the angle ξ, of the point P.

Examples of "bisymmetric" ovoid and annular bodies satisfying this assumption are represented in Figure 1.

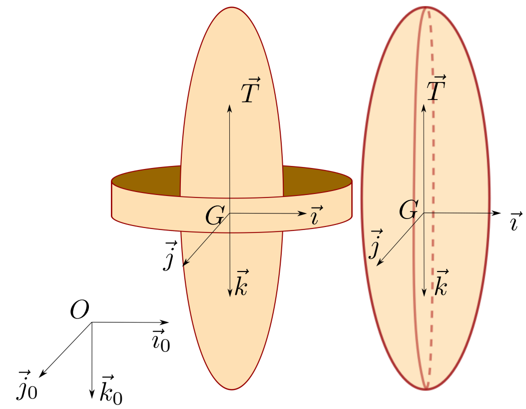

Fig. 1. Bisymmetric bodies satisfying Assumption 1.

Note that Assumption 1 implies that G is the body's geometric center.

B. Means of actuation

To cover a large number of actuation possibilities associated with man-made *underactuated* aerial vehicles, and work out general principles applicable to many of them, one must get free of actuation specificities and concentrate on operational common denominators. This leads us to assume, as in [10], that the vehicle's means of actuation consist of a thrust force $\vec{T}$ along a body-fixed direction, and a torque $\vec{\Gamma}_G$ which allows one to modify the body's instantaneous angular velocity $\vec{\omega}$ at will. In practice, this torque is produced in various ways, typically with secondary propellers (VTOL vehicles), rudders or flaps (airplanes), control moment gyros (spacecrafts), etc. The latter assumption implicitly implies that the torque calculation and the ways of producing this torque can theoretically be decoupled from high-level control objectives. The corresponding requirement is that "almost" any desired angular velocity can physically be obtained "almost" instantaneously. Under these assumptions, the control of the vehicle relies upon the determination of four input variables, namely the thrust intensity and the three components of $\vec{\omega}$. The following complementary assumption about the thrust force direction is made

Assumption 2 *The thrust force $\vec{T}$ is parallel to the axis of symmetry $G\vec{k}$, i.e. $\vec{T} = -T\vec{k}$ with T denoting the thrust force intensity.*

The minus sign in front of the equality's right-hand side member is motivated by a sign convention, also used in [10].

C. Notation

- The i_{th} component of a vector x is denoted as x_i.
- For the sake of conciseness, $(x_1\vec{\imath} + x_2\vec{\jmath} + x_3\vec{k})$ is written as $(\vec{\imath}, \vec{\jmath}, \vec{k})x$.
- $S(\cdot)$ is the skew-symmetric matrix-valued operator associated with the cross product in $\mathbb{R}^3$, i.e. such that $S(x)y = x \times y$, $\forall (x, y) \in \mathbb{R}^3 \times \mathbb{R}^3$.
- $\{e_1, e_2, e_3\}$ is the canonical basis in $\mathbb{R}^3$.

Some of the physical variables and entities used thereafter are either denoted or defined as follows.

• *m* is the mass of the vehicle, assumed to be constant for the sake of simplicity.

• $\mathcal{I} = \{O; \vec{\imath}_0, \vec{\jmath}_0, \vec{k}_0\}$ is a fixed inertial frame with respect to (w.r.t.) which the vehicle's absolute pose is measured.

• The body's linear velocity is denoted by $\vec{v} = \frac{d}{dt}\vec{OG} = (\vec{\imath}_0, \vec{\jmath}_0, \vec{k}_0)\dot{x} = (\vec{\imath}, \vec{\jmath}, \vec{k})v$.

• The linear acceleration vector is $\vec{a} = \frac{d}{dt}\vec{v}$.

• The body's angular velocity is $\vec{\omega} = (\vec{\imath}, \vec{\jmath}, \vec{k})\omega$.

• The vehicle's orientation w.r.t. the inertial frame is represented by the rotation matrix R. The column vectors of R are the vectors of coordinates of $\vec{\imath}, \vec{\jmath}, \vec{k}$ expressed in the basis of $\mathcal{I}$.

• The wind's velocity vector $\vec{v}_w$ is assumed to be the same at all points in a domain surrounding the vehicle, and its components are defined by $\vec{v}_w = (\vec{\imath}, \vec{\jmath}, \vec{k})v_w$. The *airspeed* $\vec{v}_a = (\vec{\imath}, \vec{\jmath}, \vec{k})v_a = (\vec{\imath}_0, \vec{\jmath}_0, \vec{k}_0)\dot{x}_a$ is defined as the difference between the velocity of *G* and $\vec{v}_w$. Thus, $v_a = v - v_w$.

D. Vehicle's dynamics

The external forces acting on the body are composed of the weight vector $m\vec{g}$ and the sum of aerodynamic forces denoted by $\vec{F}_a$. In view of Assumption 2, applying the fundamental theorem of mechanics yields the following equations of motion:

$$m\vec{a} = m\vec{g} + \vec{F}_a - T\vec{k}, \tag{1}$$

$$\dot{R} \ = RS(\omega), \tag{2}$$

with T and ω the system's control inputs.

III. MODELING OF AERODYNAMIC FORCES

A. Static models of lift and drag forces

The motion equation (1) points out the role of the aerodynamic force $\vec{F}_a$ in obtaining the body's linear acceleration vector $\vec{a}$. It shows, for instance, that to move with a constant linear velocity the controlled thrust vector $T\vec{k}$ must be equal to $m\vec{g}+\vec{F}_a$. It is understandable that the achievement of this equality, via the control of the vehicle, in turn involves the knowledge of $\vec{F}_a$, with its components either calculated or estimated on line from a model of this force and from other physical variables accessible to measurement. In [10], it is shown that the knowledge of this force at every time-instant allows for the design of globally stabilizing feedback controllers, *provided that it does not depend upon the vehicle's orientation*. When this latter assumption is not satisfied, as in the case where the vehicle is subjected to strong lift forces that depend on the vehicle's relative orientation w.r.t the air velocity direction, the proposed control design is invalidated. This also means that the capacity of calculating this force at every time-instant –already a quite demanding requirement– is not sufficient to design a control law capable of performing equally well in (almost) all situations. Knowing how this force changes when the vehicle's orientation varies is needed, but is still not sufficient. An original outcome of the present study is precisely to point out the existence of a generic set of aerodynamic models which allow for the design of nonlinear feedback control laws for which strong stability and convergence results can be demonstrated. Of course, the underlying assumptions are that these models reflect the physical reality sufficiently well and that the corresponding aerodynamic forces can be either measured or estimated on line.

Now, working out a functional model of aerodynamic forces from celebrated *Navier−Stokes nonlinear partial differential equations* governing the interactions between a solid body and the surrounding fluid is beyond the authors domain of expertise, all the more so that spatial integration of these equations over the shape of an object, even as simple and symmetric as an ovoid body, does not yield closed-form expressions. Notwithstanding the delicate and complex issues associated with turbulent flows –a side effect of which is the well known *stall* phenomenon– for which no general complete theory exists to our knowledge. We thus propose to take here a different route by combining a well-accepted general expression of the intensity of aerodynamic forces with geometric considerations based on the body's symmetry properties. To be more precise, let $\vec{F}_D$ and $\vec{F}_L$ denote the drag and lift components of $\vec{F}_a$, i.e.

$$\vec{F}_a := \vec{F}_L + \vec{F}_D, \tag{3}$$

with, by definition, $\vec{F}_L$ orthogonal to $\vec{v}_a$ and $\vec{F}_D$ parallel to $\vec{v}_a$. Consider also a (any) pair of angles (α, β) characterizing the orientation of $\vec{v}_a$ with respect to the body frame. The *Buckingham* $\pi-$*theorem* [12, p. 34] asserts that the intensity of the *static* aerodynamic force varies like the square of the air speed $|v_a|$ multiplied by a dimensionless function $C(\cdot)$ depending on the *Reynolds number* R_e[1], the *Mach number M*, and (α, β), i.e.

$$|\vec{F}_a| = \ k_a|v_a|^2 C(R_e, M, \alpha, \beta), \tag{4a}$$

$$k_a := \frac{\rho\Sigma}{2}, \tag{4b}$$

with ρ the *free stream* air density, and Σ an area germane to the given body shape. Then, further assuming that the direction of $\vec{F}_a$ does not (or little) depend(s) upon the airspeed $|v_a|$ and that this force does not (or little) depend(s) upon the angular velocity $\vec{\omega}$, one shows that this theorem in turn implies the existence of two dimensionless functions $C_D(\cdot)$ and $C_L(\cdot)$, and of a unit vector-valued function $\vec{r}(\cdot)$ characterizing the direction of the lift force w.r.t the body frame, such that

$$\vec{F}_L = k_a|v_a|C_L(R_e, M, \alpha, \beta)\vec{r}(\alpha, \beta) \times \vec{v}_a, \tag{5a}$$

$$\vec{F}_D = -k_a|v_a|C_D(R_e, M, \alpha, \beta)\vec{v}_a, \tag{5b}$$

$$\vec{r}(\alpha, \beta) \cdot \vec{v}_a = 0, \tag{5c}$$

In the specialized literature $C_D(\cdot)$ $(\in \mathbb{R}^+)$ and $C_L(\cdot)$ $(\in \mathbb{R})$ are called the *aerodynamic characteristics* of the body, and also the *drag coefficient* and *lift coefficient* respectively.

[1] R_e gives a measure of the ratio of inertial forces to viscous forces.

B. A subset of aerodynamic models for symmetric and bisymmetric bodies

The expressions (5) of the lift and drag forces hold independently of the body's shape, since they are derived without any assumption upon this shape. In the case of a body with rotational symmetry about the axis $G\vec{k}$ one can define $\alpha \in [0, \pi]$ as the *angle of attack*[2] between $-\vec{k}$ and $\vec{v}_a$, and $\beta \in (-\pi, \pi]$ as the angle between the unit frame vector $\vec{\imath}$ and the projection of $\vec{v}_a$ on the plane $\{G; \vec{\imath}, \vec{\jmath}\}$ (see Fig. 2). With this choice of (α, β) one has:

$$\alpha = \cos^{-1}\left(-\frac{v_{a_3}}{|v_a|}\right), \tag{6a}$$

$$\beta = \text{atan2}(v_{a_2}, v_{a_1}). \tag{6b}$$

and

$$\begin{cases} v_{a_1} = |v_a|\sin(\alpha)\cos(\beta), \\ v_{a_2} = |v_a|\sin(\alpha)\sin(\beta), \\ v_{a_3} = -|v_a|\cos(\alpha), \end{cases} \tag{7}$$

Moreover this symmetry property implies that:

P1 : the aerodynamic force $\vec{F}_a$ does not change when the body rotates about its axis $G\vec{k}$;
P2 : the aerodynamic force belongs to the plane $\{G; \vec{k}, \vec{v}_a\}$.

Property P_1 in turn implies that the aerodynamic characteristics do not depend on β, whereas Property P_2 implies that $\vec{r}$ is orthogonal to $\vec{k}$ and is independent of α. Subsequently, the expressions (5) of the lift and drag forces specialize to

$$\vec{F}_L = k_a|v_a|C_L(R_e, M, \alpha)\vec{r}(\beta) \times \vec{v}_a, \tag{8a}$$

$$\vec{F}_D = -k_a|v_a|C_D(R_e, M, \alpha)\vec{v}_a, \tag{8b}$$

$$\vec{r}(\beta) = -\sin(\beta)\vec{\imath} + \cos(\beta)\vec{\jmath} \tag{8c}$$

Observe also that, with the complementary π-symmetry w.r.t. $G\vec{\jmath}$ axis associated with bisymmetric bodies, the aerodynamic characteristics C_L and C_D must be $\pi-$periodic w.r.t. α. For low-subsonic airspeeds and small Mach numbers –typically smaller than 0.3– the dependence of the aerodynamic characteristics upon M can be neglected [12]. Furthermore, by assuming a constant Reynolds number, these coefficients only depend on the angle of attack α.

C. A class of aerodynamic coefficients yielding spherical equivalency

From the definitions of α and $\vec{r}(\beta)$, one also verifies that

$$\vec{r}(\beta) \times \vec{v}_a = -\cot(\alpha)\vec{v}_a - \frac{|v_a|}{\sin(\alpha)}\vec{k}$$

so that

$$\begin{aligned} \vec{F}_a &= \vec{F}_L + \vec{F}_D \\ &= -k_a|v_a|\left[\Big(C_L(\alpha)\cot(\alpha) + C_D(\alpha)\Big)\vec{v}_a + \frac{C_L(\alpha)}{\sin(\alpha)}|v_a|\vec{k}\right] \end{aligned} \tag{9}$$

[2]The angle of attack α so defined does not coincide with the one used for airplanes equipped with flat wings which break the body's rotational symmetry about the $G\vec{k}$ axis [1, p. 53]

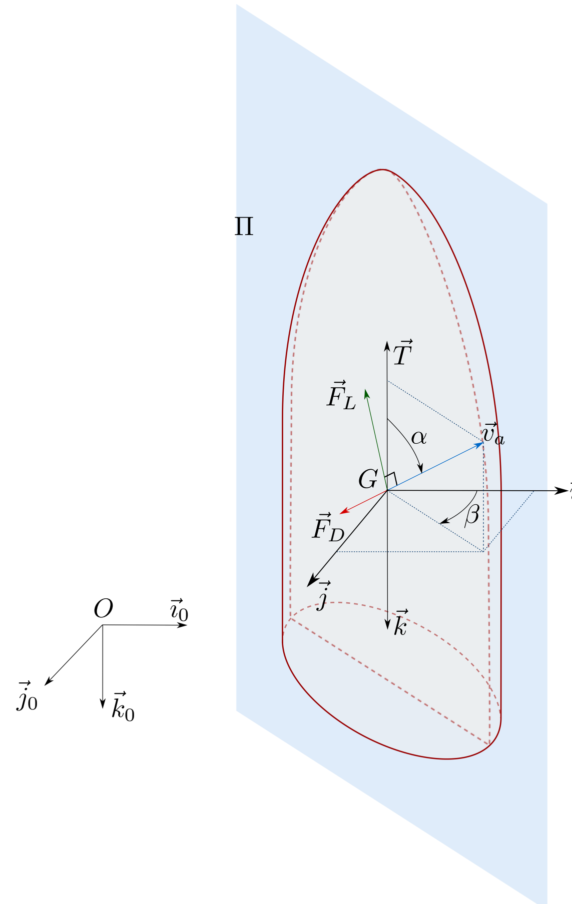

Fig. 2. Aerodynamic forces and (α, β) angles.

By using the above relationship it is a simple matter to establish the following result.

Proposition 1 *Consider a body with a rotational symmetry about one of its axes, say $G\vec{k}$, and powered with a thrust force $\vec{T} = -T\vec{k}$ parallel to this axis. Assume that the resultant of the aerodynamic forces is given by* (8) *and that the aerodynamic coefficients satisfy the following relationship*

$$C_D(\alpha) + C_L(\alpha)\cot(\alpha) = C_{D_0} \tag{10}$$

with C_{D_0} denoting a constant number. Then the body's dynamic equation (1) *may also be written as*

$$m\vec{a} = m\vec{g} + \vec{F}_p - T_p\vec{k} \tag{11}$$

with

$$T_p = T + k_a|v_a|^2\frac{C_L(\alpha)}{\sin(\alpha)} \tag{12}$$

and

$$\vec{F}_p = -k_aC_{D_0}|v_a|\vec{v}_a\ . \tag{13}$$

The important result is the non-dependence of $\vec{F}_p$ upon the angle of attack α, and thus on the vehicle's orientation. The interest of this proposition is to point out the possibility of seeing a symmetric body subjected to both drag and lift forces as a sphere subjected to an equivalent drag force $\vec{F}_p$ and powered by an equivalent thrust force $\vec{T}_p = -T_p\vec{k}$. The control design proposed in [10] can then be applied to this equivalent spherical system. The main condition is that the relation (10) must be satisfied. Obviously, this condition is compatible with an infinite number of functions C_D and C_L. Let us just point out a particular set of simple functions, already considered in the 2D-case addressed in [11], and which also satisfy the π-periodicity property w.r.t. the angle of attack α associated with bisymmetric bodies.

Proposition 2 *The functions C_D and C_L defined by*

$$\begin{cases} C_D(\alpha) = c_0 + 2c_1\sin^2(\alpha) \\ C_L(\alpha) = c_1\sin(2\alpha) \end{cases} \tag{14}$$

with c_0 and c_1 two real numbers, satisfy the condition (10) *with $C_{D_0} = c_0 + 2c_1$. The equivalent drag force and thrust intensity are then given by*

$$\vec{F}_p = -k_a(c_0+2c_1)|\vec{v}_a|\vec{v}_a \tag{15}$$

$$T_p = T + 2c_1k_a|\vec{v}_a|^2\cos(\alpha). \tag{16}$$

Note that a particular bisymmetric body is the sphere whose aerodynamic characteristics (zero lift and constant drag coefficient) are obtained by setting $c_1 = 0$ in (14). As shown in [11], the modeling functions (14) give good approximations of the physical aerodynamic characteristics measured for several symmetric wing NACA profiles, especially for small Reynold numbers yielding little pronounced stall phenomena [13].

IV. A VELOCITY CONTROL LAW ADAPTED FROM [10]

To illustrate the interest of the transformation evoked previously, let us consider the problem of stabilizing a desired (reference) velocity $\vec{v}_r = (\vec{\imath}_0,\vec{\jmath}_0,\vec{k}_0)\dot{x}_r = (\vec{\imath},\vec{\jmath},\vec{k})v_r$ asymptotically. The application of the control solution proposed in [10, Sec. III.D] to System (1)-(2), with $(\vec{F}_a, T)$ replaced by the equivalent drag force and thrust intensity $(\vec{F}_p, T_p)$ defined in Proposition 1, yields the following control expressions

$$T = \overline{f}_{a_3} + k_1|f_p|\widetilde{v}_3, \tag{17a}$$

$$\omega_1 = -k_2|f_p|\widetilde{v}_2 - \frac{k_3|f_p|\overline{f}_{p_2}}{(|f_p|+\overline{f}_{p_3})^2} + \frac{\overline{f}_p^T S(e_1)R^T\dot{f}_p}{|f_p|^2}, \tag{17b}$$

$$\omega_2 = k_2|f_p|\widetilde{v}_1 + \frac{k_3|f_p|\overline{f}_{p_1}}{(|f_p|+\overline{f}_{p_3})^2} - \frac{\overline{f}_p^T S(e_2)R^T\dot{f}_p}{|f_p|^2}, \tag{17c}$$

with $\widetilde{v} := v - v_r$, $\vec{a}_r := \frac{d}{dt}\vec{v}_r = (\vec{\imath}_0,\vec{\jmath}_0,\vec{k}_0)\ddot{x}_r$,

$$\vec{f}_a = (\vec{\imath}_0,\vec{\jmath}_0,\vec{k}_0)f_a = (\vec{\imath},\vec{\jmath},\vec{k})\overline{f}_a := m\vec{g} + \vec{F}_a - m\vec{a}_r, \tag{18a}$$

$$\vec{f}_p = (\vec{\imath}_0,\vec{\jmath}_0,\vec{k}_0)f_p = (\vec{\imath},\vec{\jmath},\vec{k})\overline{f}_p := m\vec{g} + \vec{F}_p - m\vec{a}_r, \tag{18b}$$

and $k_{1,2,3}$ three positive real numbers. Note that, using (13), the vector f_p of coordinates of $\vec{f}_p$ expressed in the fixed frame $\mathcal{I}$ is equal to $mge_3 - k_aC_{D_0}|v_a|\dot{x}_a - m\ddot{x}_r$, and is thus independent of the vehicle's orientation. Therefore, its time-derivative does not depend on the angular velocity vector ω and the above expressions of the first two components of this vector are well defined. The interest of the invoked transformation, combined with (10), lies precisely there. As for the last component ω_3, since it does not influence the vehicle's longitudinal motion due to the symmetry about the axis $G\vec{k}$, it does not have to be defined at this point. This free degree of freedom can be used for complementary purposes involving, for instance, the "roll" angle β.

Let $\widetilde{\theta} \in (-\pi,\pi]$ denote the angle between $\vec{k}$ and $\vec{f}_p$. In [10], stability and convergence properties associated with the feedback control (17) are established by using the Lyapunov function candidate

$$V = \frac{|\widetilde{v}|^2}{2} + \frac{1}{k_2 m}\big(1-\cos(\widetilde{\theta})\big).$$

More precisely, assuming that $\vec{v}_w$ and $\vec{v}_r$ are bounded in norm up to their second time-derivatives, and provided that there exists a constant $\delta > 0$ such that $|f_p| > \delta, \forall t \in \mathbb{R}^+$, one shows that the equilibrium $(\widetilde{v},\widetilde{\theta}) = (0,0)$ of the controlled system is asymptotically stable, with the domain of attraction equal to $\mathbb{R}^3 \times (-\pi,\pi)$.

Now, in practice, the control law must be complemented with integral correction terms in order to compensate for almost constant unmodeled additive perturbations. The solution proposed in [10] involves $\vec{I}_v = (\vec{\imath}_0,\vec{\jmath}_0,\vec{k}_0)I_v$ with

$$I_v := \int_0^t \dot{\widetilde{x}}(s)\,ds\ ,$$

and $\dot{\widetilde{x}} := R\widetilde{v}$ the longitudinal velocity error expressed in the inertial frame. Also, let h denote a smooth bounded strictly positive function defined on $[0,+\infty)$ satisfying the following properties ([10, Sec. III.C]) for some positive constant numbers η, μ,

$$\forall s \in \mathbb{R}, \quad |h(s^2)s| < \eta \text{ and } 0 < \frac{\partial}{\partial s}(h(s^2)s) < \mu.$$

It then suffices to replace the definitions (18) of $\vec{f}_a$ and $\vec{f}_p$ by the following ones

$$\vec{f}_a := m\vec{g} + \vec{F}_a - m\vec{a}_r + h(|I_v|^2)\vec{I}_v \tag{19a}$$

$$\vec{f}_p := m\vec{g} + \vec{F}_p - m\vec{a}_r + h(|I_v|^2)\vec{I}_v \tag{19b}$$

in (17) to obtain a control which incorporates an integral correction action and for which strong stability and convergence properties can also be proven (more details in [10]).

V. CONCLUSION AND PERSPECTIVES

The paper sets basic principles for the modeling and nonlinear control of aerial vehicles subjected to strong aerodynamic forces. Possible extensions are numerous. They concern in particular airplanes and other vehicles whose lift properties mostly rely on the use of large flat surfaces (wings) which breaks the body symmetries here considered. The dreaded stall phenomenon, when it is pronounced, is not compatible with transformations alike these evoked in the paper, because it forbids the existence of equivalent drag forces that do not depend on the vehicle's angle of attack. Nor is it even compatible with the uniqueness of cruising equilibria and the objective of asymptotic stabilization, as pointed out in [14]. Its importance at both the modeling and control levels, and its consequences during transitions between hovering and lift-based-cruising, need to be studied and, if possible, attenuated via an adequate control design. Clearly, the control solution here proposed also calls for a multitude of complementary extensions and adaptations before it is implemented on a physical device. Let us just mention the production of the control torque allowing for desired angular velocity changes and the determination of corresponding low level control loops that take actuators' physical limitations into account –in relation, for instance, to the airspeed dependent control authority associated with the use of flaps and rudders. Measurement and estimation of various physical variables involved in the calculation of the control law, other than the ever needed information about the vehicle's position and attitude, such as the air velocity and the angle of attack, or the thrust force produced by a propeller, also involves a combination of hardware and software issues which are instrumental to implementation.

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